Christoph Selter & Martin Bonsen; June 2014
Christoph.selter@math.tu-dortmund.de

The PIKAS project

1 Conception

1.1 Project approach

PIKAS is a cooperation project between the universities of Dortmund and Münster, the Ministry of Education of NRW (North Rhine-Westphalia) and the "Deutsche Telekom Stiftung" (Deutsche Telekom Foundation). It is regarded as a successive example for public-private-partnership. The project focuses on realising the standards of school mathematics named by the Kultusministerkonferenz (Conference of Cultural Ministers) and on the interplay named in the curricula between **p**rocess-related competencies (also named 'general mathematical competencies') and content-related competencies (**I**nhaltsbezogene **K**ompetenzen) based on the encouragement (**A**nregung) of subject related **S**chool development (abbr.: **PIKAS**).

Other important topics are dealt with as well (see 3.1). It is aimed for an education that is geared to the children's individual competences and deficits as well as their different experiences and interests. Moreover it is aimed to encourage children to more and more develop their potential and motivation with self-responsibility, to activate their interest and mathematical self-conception and to arouse - or even better: to nourish - the spirit of research in mathematical contexts.

The sub-project PIK mainly aims to present assistance and to develop supportive materials for the various people that are involved in developing competence-orientated mathematics education further (see 3.3).

The sub-project AS offers support for the subject-related development of schools. Therefore AS is mainly addressed to headmasters and people – like a school's math experts - who are dealing with the development of mathematical education of schools and school-networks.

The ideas and materials are published on the project website www.pikas.tu-dortmund.de and are available for free for everyone who is interested. They can be used for modifying one's own teaching in classes or for (self-) in-service teacher training.

PIKAS concentrates on primary schools (grades 1 through 4). Still many ideas and suggestions are relevant and can be used - maybe after a few modifications - for preschool education and the lower years of secondary schools. The transitions from preschool to primary school and from primary to secondary school are explicit topics of PIKAS.

One important point of reference is the mathematics curriculum for primary schools in NRW. Anyway the project was designed so that the materials can be used to their full extend in any other state. The statistics about how often the website was accessed (19,000 accesses to the home page per month) and getting many requests for project events from outside of NRW show how intense the PIKAS offers are used.

1.2 Competence-oriented mathematics teaching

Both, the standards of education (named by the KMK) and the different mathematics curricula for primary schools of the various states assume that learning mathematics is more than only acquiring *knowledge* (like knowing all results of the multiplication table by heart) or acquiring skills (like knowing how to use the standard algorithms for addition). Next to those content-related competencies the process-related competencies like 'reasoning' and 'presenting results' are aimed to be developed in mathematic teaching.

The daily experiences in schools and empirical studies show that process-related competencies cannot emerge by themselves. Not only the children but also the teachers have to be supported. The support for German teachers is necessary as they often did not get trained (with the necessary extend) in mathematics or their training dates back a few decades.

Therefore the development of education and in-service teacher training have been implemented in PIKAS from the beginning on: The teachers are supported to be able to help their students to achieve the expected competencies named in the curriculum and in the standards for school mathematics. To achieve this, teachers were part of the project from the beginning onwards in developing and evaluating the material. Practical experience on the one hand and the theoretical distance of science on the other hand have complemented and influenced each other in this way.

1.3 Subject-related school development through professional learning communities

Research studies on subjective theories have shown that teachers develop their own basic attitudes that mark their everyday teaching and behaviour. If you want the theoretical development of education to really be implemented in the everyday teaching, you have to not only consider the professional knowledge but also the attitude that determines the teaching.

To acquire the new knowledge works best for those teachers who can find a connection between their current attitude and knowledge on the one side and the new ideas on the other side. This gets easier when the in-service teacher training is long-term orientated, if the teachers can participate in choosing topics, if training and practical experiences at school alternate and if the teachers work together in small groups. Therefore the development of education within the framework of PIKAS is most effective when the teachers are working and professionalizing in teamwork and within given structures of the individual school. Not least to improve the transfer and sustained yield of traditional in-service training it is reasonable to integrate teachers in cooperative teamwork with a long-term perspective.

Doing professional teamwork, teachers work together to improve and develop the education of their subject further. Characteristics of such teamwork are reflecting the education together, to open up one's own classes for colleagues, to have the same basic understanding of goals and to focus on improving the learning process for all children.

1.4 Structure of the project, governance and time management

The project work currently is organised by three teachers named by the ministry of education - the so

called PIKAS-Teachers - who spend 50% of their work time with PIKAS. They get help from three research assistants who are employed by the Telekom Foundation or rather the contributing universities. The cooperative project was designed and initiated by Christoph Selter. The administration work is cooperatively divided between the professors Selter, Bonsen and Bos. Strategic decisions are made by the management committee consisting of the administrators, two members of the ministry of education and two members of the Telekom Foundation. The committee meets four times a year.

The project is arranged in three stages. The **development stage** took three years (from February 2009 until January 2012). In this stage all the conception work has been done as well as the biggest part of developing, testing and improving the material. The **expansion stage** (from February 2014 until July 2014) was mainly about expanding the project in cooperation with the five District Councils and with 21 of 53 local education authorities in NRW. The conception work and the development of material were terminated during this stage as well. Now the **implementation stage** (beginning in July of 2014) will be used for steadying and expanding the cooperation with (other) local education authorities in NRW. It is planned to set up cooperation with partners in different states than NRW as well.

2 Basics

2.1 Conception basis

The PIKAS project is understanding mathematics as a science of patterns (Wittmann & Müller 2012). Different orders, structures, relations, connections, peculiarities, dependencies or regularities can be explored, continued, developed and created. Mathematics is an activity - something one does (Freudenthal 1982).

Mathematical objects have a theoretical nature (Steinbring 1999). For example: The number 5 does not exist for itself in nature, but its representatives do. There are representatives that do not exist in everyday life as well - like for example the five in a number line or five reversible counters. In this respect it is necessary to also implement the theoretical represents of numbers and not only use the meaningful mathematics of everyday life in order to not miss any learning opportunities. Winter (1995, page 37) names three basic experiences that mathematical education has to offer and to connect with each other:

- "To notice and understand occurrences - that come up in the everyday life concerning nature, society and culture and are or should be meaningful to anyone - in a specific way
- To learn and understand mathematical objects and facts as a unique and deductively ordered world, that can be represented with language, symbols, pictures and formula
- To acquire abilities in problem solving that can be used for more that mathematics (heuristic abilities) by dealing with tasks."

This comprehensive summary about the educational goal is the frame of reference for PIKAS. It is the basis for the standards for school mathematics defined by the KMK (2004, 2005 and 2012) that all

children have to have achieved at a certain age. As mentioned above, the named standards differentiate between **content-related competencies** (like 'Numbers and Mathematical Operations' or 'Space and Form') and general mathematical competencies, that - in mathematic-didactical discourse - are often called process-related competencies (like 'Reasoning' or 'Modelling'; see e.g. Walther, Selter & Neubrand 2008 or Granzer, Köller, Reiss et al. 2008).

Even though it is very important to enforce processes like 'Exploring', 'Discovering', 'Describing' or 'Reasoning' in mathematical education for primary schools, it is essential to know that mathematical classes consist of practice-of-skills-times as a big part. In this context it is one main goal of PIKAS to spread the concept of 'Productive Practising'. The concept is about giving children the chance to practice as well as to train their process-related competencies by giving them rich mathematical tasks like 'packages with a pattern' (Müller, Selter & Wittmann 2012, Leuders 2009).
For lack of space we cannot go into the conceptual and specific basis concerning the PIKAS materials. The material is differentiated into ten different 'houses' concerning topics like arithmetic difficulties or performance reviews (see 3.1).

2.2 Teaching and learning research

Findings from the teaching and learning research have been implemented into the PIKAS-concept and can be bundled into three main points.

Active-discovery learning: The PIKAS-concept regards children as active learners (Reinmann-Rothmeier & Mandl 2001). The children achieve the competences by self-consistently helping to organize their learning-progress and by integrating new competencies into the existent knowledge structure and interpreting them against the background of their individual experiences. Moreover learning is a social process as achieving competencies takes always place in interaction with others. After all learning is a situational process, because which competencies can be achieved is depending on the specific context and situation (Gräsel & Parchmann 2004a). To offer many and effective learning incitements is very important to help children in developing mathematical competencies. Children use and shape the learning incitements. Against this background it is very important to coordinate and match the learning preconditions to the learning incitements (Siegler, De Loache & Eisenberg 2005; Weinert, Doil & Frevert 2008).

Support based on diagnoses: As there is a very big variety and heterogeneity between all pupils (Prenzel & Burba 2006) principles of 'Diagnoses and individual support' (Diagnose und individuelle Förderung, short DiF) have become more and more important for education-political, didactical and theoretic-professional discussions and developments (see Becker et al. 2006). Studies have shown that teaching-/learning-processes can be effective and permanent if they take the child's individual learning precondition as a basis and try to develop them further in an adaptive way (Helmke 2012). There is no conflict in supporting autonomy on the one hand and goal-orientation on the other hand. Quality-education - like it is demanded by PIKAS - lives of the productive tension between openness and structure.

Challenging learning environment: Putting all quality criteria of mathematical education together that can be found in specialist literature (see e.g. Helmke 2012, Seidel & Shavelson 2007; Baumert et al. 2004) one can identify three different criteria that are continuously discussed on the PIKAS-project:
(1) a composition of lessons that is rich of content (with many possibilities to achieve competencies and to link topics up that are inner-mathematic and extra-mathematic as well)
(2) a permanent cognitive activation of the learner and
(3) an effective and student-orientated classroom management.

2.3 School Development Research

The points in which good, innovative schools differ most from not so good and innovative schools are class-related conversations, reciprocal class-supervision and reviews, developing teaching materials together and all teachers trying to improve one another's didactical competencies (Little 1982). Studies have shown that high quality teaching leads to good learning outcomes. Characteristics that lead to high quality teaching turned out to be teachers who get supported by their principal during their professional development and teachers who have a positive cooperation and relation to their colleagues (Bryk et al. 2010; Prenzel, Friedrich & Stadler 2008). The approach of "Professional Learning Communities" (Lomos, Hofman & Bosker 2011) is a now well probed scheme for an effective development of education. Typical strategies for developing lessons within professional learning communities are teaching-related cooperation (plan, prepare and evaluate lessons), reciprocal class-supervision, deliberating dialogues about teaching as well as attending new teachers (Bonsen, Hübner & Mitas 2013). PIKAS uses the concept of "Professional Learning Communities" as a model for development of lessons within the school. The concept is used in in-service teacher training as an encouragement for subject-related development of lessons, too.

2.4 Research about in-service teacher training

The following five central themes based on Desimone (2009) or Lipowsky and Rzejak (2012) are the basis for the PIKAS concept of in-service teacher training.
Participant-orientation: In-Service should take the participant's individual preconditions as a basis for developing their competencies goal-orientated and demand-orientated with regard to concrete tasks (Krainer 1998). Ideally the trainings are not one-sided but participative and give all participants the chance to a self-dependant arrangement and performance (Selter 2006).
Stimulation of ooperation: Participants should work together to solve problems theoretically and to implement the newly learned theories into everyday life. In this way the cooperation between all participants is stimulated and creates the basis for a sustainable cooperation (Garet et al. 2010; Kennedy 1999).
Case-orientation: Everyday situations ('cases') taken from classes are the starting point as well as the area of application for teaching and learning within the context of the project. Especially the orientation regarding to the practical experiences of all participants is the most important point of reference for organising the in-service training (Timperley et al. 2007).

Diversity of methods: PIKAS training is open for the participants to get actively involved, to experience self-efficiency and to transfer the new knowledge (Carpenter et al. 1989). Having the mix of diverse methods during the presence phase, self-studies, collaborative work in pairs or small groups and e-learning phases make a good combination of input-, trial- and deliberation-phases (Kennedy 1999).

Deliberation training: All participants are encouraged to deliberate together about subjects that were dealt with and about themselves and their own teaching in classes (Putnam & Borko 2000, Boyle et al. 2005).

3 Concretion

3.1 Key topics

As it was said in 1.1 PIKAS wants to support teachers in setting up an education that is up to time - an education that understands children to be active learners, that is based on the individual academic level and learning opportunities each child has. Mathematical education is aimed to be competence-orientated and teaching should use mathematical structures and connections to the reality as a meaningful context of learning-development.

The key topics of the PIKAS-project are represented by the 10 "houses" (H1 to H10). Each "house" contains materials of one key topic concerning development of lessons (http://pikas.tu-dortmund.de/pik). It is aimed for mathematical lessons, that...

- train process-related competencies as well as content-related competencies (H 1)
- sustainably build up competencies beginning at preschool and leading to secondary schools (H 2)
- integrate prevention, diagnoses and training concerning calculation difficulties into everyday classes (H 3)
- consider the training of language skills as an elementary part of mathematical education (H 4)
- find a balance between individual ways of thinking and expected competencies (H 5)
- make use of the heterogeneous academic level between pupils by applying concepts like the 'natural differentiation' (H 6)
- get the pupils to solve fruitful and challenging tasks instead of just keeping them occupied (H 7)
- give the pupils a chance to help forming their learning process actively and self-sufficiently (H 8)
- consider continuous and potential-orientated testing of the academic level as the essential basis for individual support (H 9)
- make use of process-orientated performance reviews and dialogic performance feedback (H 10)

The framework of the KMK standards for school mathematics and actual curricula as well as findings from current technical didactics, educational research and primary school pedagogy were concretised

in these ten key topics. Furthermore there were - especially during the start-up period - various expert consultations (teachers, propagators) for surveying the support requirements.

3.2 Realisation

The sub-project PIK has developed three different types of material that are all connected to each other. By now the development of the material is finished with only a few exceptions. The material is available for free on the website. A 'Guided Tour' informs potentially interested people within five minutes about the site's structure and possibilities of use (http://pikas.tu-dortmund.de/141).

The **materials for in-service training** consist of background information, Power Point presentations, moderation guidelines and materials for participants. They deal with topics like preventing calculation difficulties, productive handling of heterogeneity in primary schools or evaluation of process-related competencies. These materials can on the one hand be used and modified by moderators who conduct in-service trainings. On the other hand it is possible as well for teachers to use the website for self-education. The material is structured into 35 modules.

The **materials for teaching** - like factual information, lesson-planning, work-sheets, mathematic posters concerning 54 learning environments - are available as a download on the website as well. The material developed by the PIK-team was tested within school projects and continuously developed further.

The **information material** makes it possible to use the website for self-education by presenting texts and links concerning different subject-related, didactical topics. More over the project created 16 information videos, that present in which ways a competence-orientated mathematical education can be realised. The films are high quality and addressee-orientated (http://pikas.tudortmund.de/152). Materials for the work with parents concerning the different mathematic fields are provided on the website as well (like information letters for parents or advisory texts).

The sub-project AS has developed information material and inspirational material for headmasters, specialised groups and other people who are interested in developing mathematical classes. Topics like "Cooperational Work Shadowing" and "Professional Learning Communities" are for example illustrated by film documentations about cooperative planning, realising and deliberating lessons in a team (http://pikas.tu-dortmund.de/159).

The transfer into practical education and in-service training is a key characteristic of PIKAS. This means that the empirically-based development of materials is implanted into a global transfer-concept. This concept is adumbrated in section 3.

3.3 Systemic development of education through (self-) education of teachers

The project is based on a systemic view on developing mathematical classes: The aim is to support different people involved by presenting conceptions and material for classes and advanced trainings.

One main target audience are **Teachers**. They can use PIK-teaching materials and develop their professional competence further by planning, realising and deliberating mathematical classes. On the

one hand teachers can benefit from the information material and the material for advanced training on the website and on the other hand they can benefit from training seminars that were prepared with support from PIKAS (http://pikas.tu-dortmund.de/156). Also people who have not been trained in mathematics or whose training dates back a few years can benefit from low-threshold offers (like videos about mathematics teaching, documented examples from classes, suggestions that are easy to realise…).

Another main target audience are **Teacher teams** as **School management** or **Heads of professional groups**. They make development of lessons more popular within their colleagues and work on topics like "Cooperation", "Work Shadowing of Colleagues" or "Leadership and Guidance" (http://pikas.tu-dortmund.de/as).

A third main target audience is **Multipliers** (e.g. expert advisors, advanced trainers, school supervisors or people who train teachers). PIKAS offers these people elaborated modules (see above), that are free for anyone to use for their own purposes.

Development of teaching only works well, if **Parents** (as well as everyone interested from the public) can be seen as "allies". Therefore PIKAS has developed materials for parents (http://pikas.tu-dortmund.de/129). Moreover teachers can find additional material that can be supportive with for example parent-teacher-conferences (see e.g. http://pikas.tu-dortmund.de/157).

Even though they usually do not come into contact with the PIKAS-material, another main target audience are **Pupils**, who can benefit from PIKAS. It is contemplated to establish a PIKAS-Pupils-Website with concrete materials for children but at this very moment it is not practicable.

Overall one can say that PIKAS offers a framework containing exemplarily materials for teaching, advanced trainings and for information. This means that PIKAS does not substitute the leading medium of classes (usually the schoolbook) nor the preservice teacher-training. Instead PIKAS provides the basis for deliberating classes and advanced trainings. The trainings can be very individually arranged with the help of PIKAS-materials.

At the very moment expert staff from the field of preschool education as well as staff from all-day schools are no primary main target audiences for PIKAS.

4 Evaluation and Transfer

4.1 Multiple interplays between development and evaluation

All materials have been developed in close cooperation between mathematics educators, educational researchers, PIKAS-teachers and teachers of the project schools. Generally the materials were tested and reworked twice or even triply.

The rework of the materials was based on notes taken from classes or advanced trainings, from analysing the pupil's written documents or the evaluation of questionnaires and from debriefings between everyone involved after every testing. The information material was tested multiply as well by giving it to (potential) receivers and asking them for a critical feedback.

4.2 Evaluation of the project

Even though the "Four levels of effectivity" (see Lipowsky) are technically speaking only meant for advanced teacher trainings and measures for professionalisation whereas PIKAS is set up on a wider basis, in the following the four levels will help to structure the evaluation.

Evaluation received from the receivers: All advanced trainings were evaluated regarding their advancements by giving out questionnaires, having feedback rounds with all participants and collegial work-shadowing. The utilisation of the materials for teaching and the information materials were evaluated during the development stage through phone-interviews with all cooperation schools. In the context of the evaluation-questionnaire 2011 (see below) the teachers had the chance to evaluate in how far the PIK-materials are supportive. The results show that the project has a positive impact on the education in schools.

Expanding the teacher's knowledge: Moreover there were two standardised samplings. The first survey in 2010 was completed by 1502 teachers and 208 school managements from 219 different primary schools. The second survey in 2011 was only open to those who had been part of the first one. 806 teachers from 180 different primary schools answered the second survey. Additionally 58 teachers from PIKAS cooperation schools completed the survey. One result of the survey was that teachers from cooperation schools had better data concerning the development of mathematic classes than teachers from the other schools. This is a sign for successful project work and that the curriculum was implied successfully as well. In the context of the BMBF-project (project of the Federal Ministry for Education and Research) 'Advanced training of teachers for supporting innovations of mathematical classes' (short LIMa) teachers of 25 primary schools were tested about how their knowledge developed through training concepts like PIKAS. First results have shown that PIKAS was successful.

Acting in everyday classes: Because of the available capacities so far this level was only examined in two qualification works that are not finished yet.

Effects on pupils: Suitable data of around 2000 third- and fourth-graders have been collected. The evaluation of the data is not completed yet. It is planned to collect more data about PIKAS-classes when the competencies of all fourth-graders are tested in 2015 on the occasion of TIMSS.

4.3 Implementation into teacher-training

The PIKAS-materials are an integral part of teacher-training at the universities of Dortmund and Münster. University wide evaluations show that PIKAS is successful in being accepted as well as regarding the learning development of the students. Feedback and requests for video-passwords show that several colleagues out of the preservice training phase from NRW (and more regions, even in Austria and Switzerland) have used the material for their training. More over many trainers out of the second training phase use PIKAS-materials. Here as well the usage is not limited to NRW only.

4.4 Implementation into the advanced teacher training and transfer concept NRW

The project PIKAS has been active in different areas of advanced training from the very beginning of the **development phase** (2009-2012) and still is (http://pikas.tu-dortmund.de/157). It has reached about 15.000 persons.

Now the **expansion stage** (2012-2014) is meant to enforce the cooperation with the five District Councils and with 28 of 53 local education authorities in NRW. Primary it is about supporting and developing structures and concepts that contribute to developing mathematic classes in primary schools further. An integral part of the cooperation is the usage of PIKAS-materials (in teacher teams) for development of lessons. As obviously there is more useful materials than only the ones from PIKAS it is one main goal to coordinate and focus available resources. In this way it is possible to be on a sustainable way for successful development of lessons (for an overview, see http://pikas.tu-dortmund.de/163).

The implementation stage (from 2014 onwards) now is about building up a "Network for mathematical classes at primary schools in NRW". The network is meant to connect different levels (NRW, District Council X, Education Authority Y) and their players, as they are all responsible for developing mathematic classes further (teacher-trainers in all three stages, members of the expert teams for teacher-trainings, Ministry, District Councils, directors of Education Authorities, headmasters, teachers etc.).

4.5 Transferring beyond NRW

The project is open for further cooperation to schools that are not in NRW at the extent of available resources. All PIKAS-materials and presentation-offers are the basis for the work of the 'Deutsches Zentrum für Lehrerbildung Mathematik" (DZLM, German Centre for Mathematic Teacher-Training). The PIK-offers are used intensively for cooperation-projects with primary schools. The AS-offers are the basis for the DZLM work-field 'Enhancement of cooperative Competence-development respectively cooperative Work-shadowing'.

Members of PIKAS have introduced the project to colleagues from the federal states Bavaria, Baden-Württemberg, Hessen, Schleswig-Holstein and Thuringia as well as to colleagues from abroad (Austria, Switzerland). The project was introduced through academic lectures, qualification courses for propagators or within the framework of advanced teacher-trainings. Some persons are advised at their location for more support.

Moreover PIKAS is active in the field of 'Marketing'. Currently every three months there is a PIKAS-newsletter sent to 1,800 subscribers. The newsletter informs about the new developments within the project.

4.6 Transfer the concept and use it for other subjects

The project approach, many topic-related concepts and materials are meaningful for the education of

other subjects than mathematics as well. The materials are either transferable so they can be used or sometimes even directly usable. For example PIK-concepts and materials for promoting language skills in the context of classes, for individual and cooperative learning or for performance reviews are easy transferable. Moreover findings and products of the sub-project AS regarding topics like 'Leadership and Guidance', 'Cooperation' or 'Feedback and Work-Shadowing' (Professional Learning Communities) can be transferred and used for other subjects as well.

5 Publication of teaching- and learning-materials

All materials are available on the website as a download free of charge. Only information videos (http://pikas.tu-dortmund.de/152) are protected by a password for legal reasons. Anyway the passwords were sent to all primary schools of NRW and also given to anyone requesting them and being active in (advanced) teacher-training. (In case you would like to have a look at some videos: username: schulenrw; password: maruta42.)

You can find an overview of all materials of the sub-project PIK by following the URL http://pikas.tu-dortmund.de/pik. An overview over all AS-materials can be found via http://pikas.tudortmund.de/as.

It is important to know that the materials from both sub-projects PIK and AS are closely related to each other. Sometimes they have been developed in close cooperation and then published on one of the sites, like for example the materials concerning 'Quality criteria for good mathematic education' or 'Cooperative work shadowing' (http://pikas.tu-dortmund.de/159).

The PIK-website offers an overview of available materials for teaching, materials for advanced trainings and information materials under the named URL. On top of that there are overviews of all information videos, parent information and information about the topic 'Handling of Heterogeinity'.

The publishing house Cornelsen has released the book 'Mathe ist Trumpf' (Math is Trumps') in the beginning of 2013. It contains selected PIKAS-materials for teaching and was send to all primary schools of NRW free of charge. More exemplars are available for the factory costs of 10 Euros at Cornelsen. Diverse project-publications were released in magazines and books (http://pikas.tu-dortmund.de/158).

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